Evaluate $\int_4^\infty \frac{1}{(\sqrt{x}-1)^3}\,dx$. $= \lim_{N\to\infty} \int_4^N \frac{1}{(\sqrt{x}-1)^3}\,dx$

SCORE: _____ / 7 PTS

$U=\sqrt{x}-1$

$x=(U+1)^2$

$dx=2(U+1)\,dU$

$= \lim_{N\to\infty} \int_1^{\sqrt{N}-1} \frac{2(U+1)}{U^3}\,dU$

$= \lim_{N\to\infty} \int_1^{\sqrt{N}-1} \left(\frac{2}{U^2}+\frac{2}{U^3}\right) dU$

① EACH

$= \lim_{N\to\infty} -\left(\frac{2}{U}+\frac{1}{U^2}\right)\Big|_1^{\sqrt{N}-1}$

$= \lim_{N\to\infty} -\left(\frac{2}{\sqrt{N}-1}+\frac{1}{(\sqrt{N}-1)^2}\right)+3$

$= 3$

Evaluate $\int x \ln(x^2+4x+13)\,dx$. $= \frac{1}{2}x^2\ln(x^2+4x+13) - \int \frac{x^3+2x^2}{x^2+4x+13}\,dx$

SCORE: _____ / 9 PTS

u		dv
$\ln(x^2+4x+13)$	$+$	x
$\frac{2x+4}{x^2+4x+13}$	$-$	$\frac{x^2}{2}$

$$\begin{array}{r} x-2 \\ x^2+4x+13 \enclose{longdiv}{x^3+2x^2} \\ \underline{x^3+4x^2+13x} \\ -2x^2-13x \\ \underline{-2x^2-8x-26} \\ -5x+26 \end{array}$$

$$= \frac{1}{2}x^2\ln(x^2+4x+13) - \int\left(x-2+\frac{-5x+26}{x^2+4x+13}\right)dx$$

$$= \frac{1}{2}x^2\ln(x^2+4x+13) - \int\left(x-2+\boxed{\frac{-\frac{5}{2}(2x+4)+12(3)}{(x+2)^2+9}}\right)dx$$

$1\frac{1}{2}$

$$= \frac{1}{2}x^2\ln(x^2+4x+13) - \frac{1}{2}x^2 + 2x$$

$\frac{1}{2}$

$$+ \frac{5}{2}\ln\left|x^2+4x+13\right| - 12\tan^{-1}\frac{x+2}{3}$$

$+C$

1 EACH

UNLESS OTHERWISE NOTED

Evaluate $\int \frac{23-2y}{(y-1)(y^2+y-2)}\,dy$. $= \int \frac{23-2y}{(y-1)^2(y+2)}\,dy$

SCORE: _____ / 7 PTS

$$= \int \left(\frac{A}{y-1} + \frac{B}{(y-1)^2} + \frac{C}{y+2} \right) dy$$

$$\boxed{23-2y = A(y-1)(y+2) + B(y+2) + C(y-1)^2}$$

$y=1: \; 21 = 3B \rightarrow B=7$

$y=-2: \; 27 = 9C \rightarrow C=3$

COEF OF y^2: $0 = A+C$

$A = -C = -3$

$$= \int \left(\boxed{\frac{-3}{y-1}} + \boxed{\frac{7}{(y-1)^2}} + \boxed{\frac{3}{y+2}} \right) dy$$

$$= \underbrace{-3\ln|y-1|}_{\frac{1}{2}} \underbrace{- \frac{7}{y-1}} + \underbrace{3\ln|y+2|}_{\frac{1}{2}} + C$$

1 EACH
UNLESS OTHERWISE NOTED

SANITY CHECK: $y=2$

$$\boxed{\begin{aligned} LHS &= \frac{19}{4} \\ RHS &= -3 + 7 + \frac{3}{4} = 4\frac{3}{4} = \frac{19}{4} \checkmark \end{aligned}}$$

Evaluate $\int_{-\pi}^{\pi} \tan^2 x \sec^4 x \, dx$. $= \int_{-\pi}^{-\frac{\pi}{2}} \tan^2 x \sec^4 x \, dx + \int_{-\frac{\pi}{2}}^{0} \tan^2 x \sec^4 x \, dx$

SCORE: ____ / 7 PTS

$$+ \underbrace{\int_{0}^{\frac{\pi}{2}} \tan^2 x \sec^4 x \, dx}_{} + \int_{\frac{\pi}{2}}^{\pi} \tan^2 x \sec^4 x \, dx$$

DIVERGES

$$= \lim_{N \to \frac{\pi}{2}^-} \int_0^N \tan^2 x \sec^4 x \, dx$$

$u = \tan x \to x = \tan^{-1} u$

$du = \sec^2 x \, dx$

① EACH

$$= \lim_{N \to \frac{\pi}{2}^-} \int_0^{\tan N} u^2(u^2+1) \, du$$

$$= \lim_{N \to \frac{\pi}{2}^-} \int_0^{\tan N} (u^4+u^2) \, du$$

$$= \lim_{N \to \frac{\pi}{2}^-} \left(\frac{1}{5}u^5 + \frac{1}{3}u^3\right)\Big|_0^{\tan N}$$

$$= \lim_{N \to \frac{\pi}{2}^-} \left(\frac{1}{5}\tan^5 N + \frac{1}{3}\tan^3 N\right)$$

$$= \infty$$